

SAMPLE PROBLEMS - MIDDLE SCHOOL PRAXIS II

1. Let $S = \{W, X, Y, Z\}$ and let $\$$ be a binary operation defined on S according to the following table:

$\$$	W	X	Y	Z
W	X	Z	W	Y
X	Z	Y	X	W
Y	W	X	Y	Z
Z	Y	W	Z	X

What is the inverse of Z?

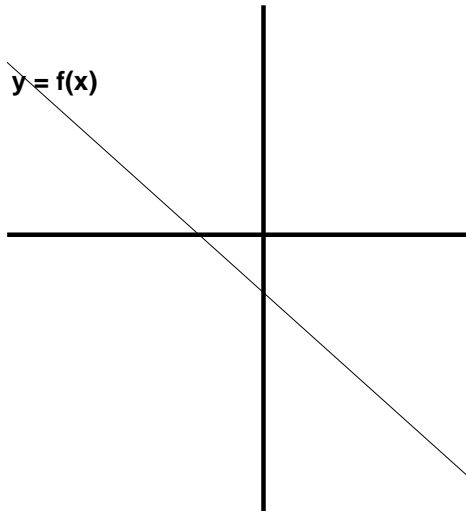
- A. W
- B. X
- C. Y
- D. Z does not have an inverse.

2. Which of the following expressions is equal to

$$\frac{(x/3) - (3/x)}{1 - (3/x)}$$

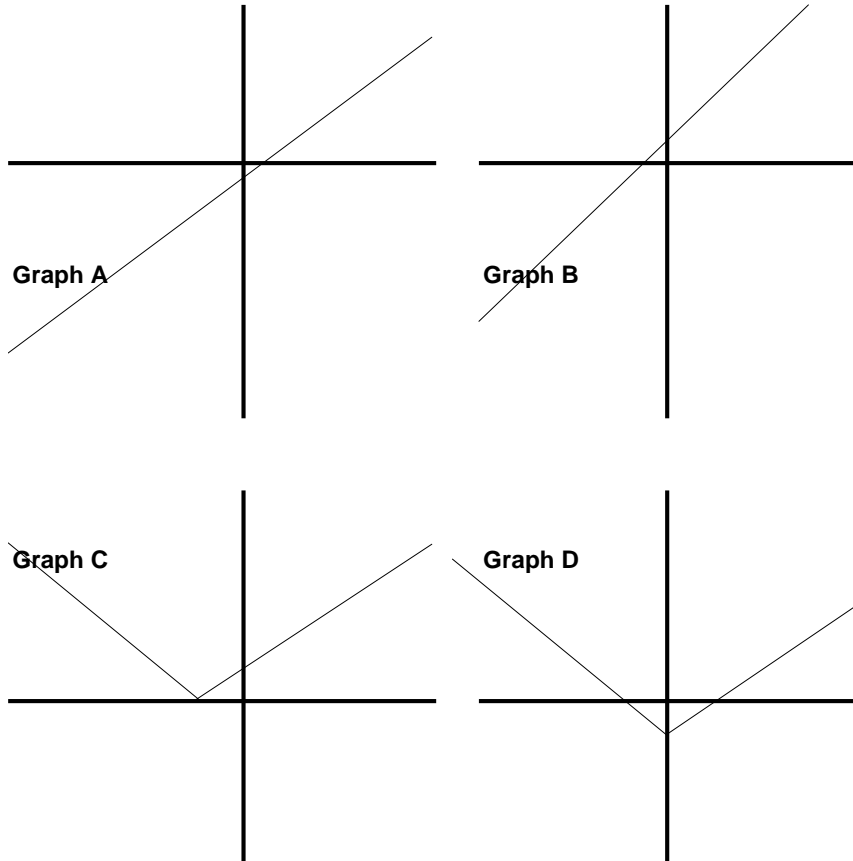
- A. $(x - 3)/3$
- B. $(x + 3)/3$
- C. $3(x - 3)$
- D. $3(x + 3)$

3. The following is a graph of $y = f(x)$:



Which of the following graphs represents $|f(x)|$?

- A. Graph A
- B. Graph B
- C. Graph C
- D. Graph D



4. Calculate $i^5 * i^6 * i^7 * i^8$.

- A. +1
- B. -1
- C. +i
- D. -i

5. Which of the following equations provides the closest fit to the following data?

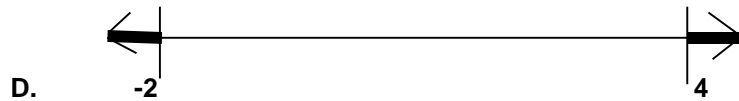
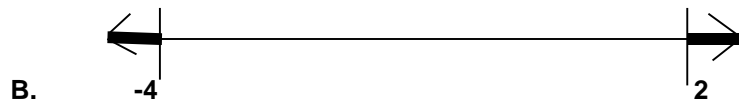
x	1	2	3	4	5
y	10	5	2.5	1.25	0.625

- A. $y = -10*(x - 1) + 10$
- B. $y = 10/x$
- C. $y = 1.25*x^2 - 8.75x + 17.5$
- D. $y = 20/2^x$

6. A homeowner puts his house up for sale at a price of \$250,000 on January 1. For each month that the house does not sell, he reduces the price 0.5%. If the house is still unsold at July 1, approximately what is the price on that date?

- A. 150,000
- B. 180,000
- C. 210,000
- D. 240,000

7. Which graph represents the solution of $|3x + 3| \geq 9$?



8. Find the volume of a pyramid having slant height 5 and a base that is a square with side 8.

- A. 64
- B. 106.67
- C. 320
- D. 576

9. Solve for x in $4^{|x+1|} + 3 = 19$

- A. +1
- B. -3 and +1
- C. -1 and +3
- D. +1 and +3

10. (Extended Response) The set $P = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 \}$ has mean = 5.5 and standard deviation = 2.9 (rounded).

A. Consider the following experiment: Five elements of P are chosen at random without replacement to create a subset A_1 of P . How many different subsets can be created?

B. Suppose the experiment above is performed 100 times, creating 100 subsets A_i (not necessarily all different). For each A_i , the mean \bar{x}_i of the five numbers is calculated. Now suppose the mean and standard deviations of these 100 \bar{x}_i numbers is calculated. What would you expect the mean of the 100 \bar{x}_i numbers to be? What would you expect the standard deviation of the 100 \bar{x}_i numbers to be?

C. Instead of 100 times, the above experiment was performed five times, producing the following five subsets:

$$A_1 = \{ 5, 4, 3, 8, 9 \}$$

$$A_2 = \{ 10, 2, 7, 4, 8 \}$$

$$A_3 = \{ 7, 10, 5, 1, 4 \}$$

$$A_4 = \{ 9, 2, 3, 7, 5 \}$$

$$A_5 = \{ 1, 8, 6, 2, 10 \}.$$

Calculate the mean \bar{x}_i of each subset A_i ; then calculate the mean and standard deviation of the five \bar{x}_i numbers.

D. What principle or theorem is this experiment trying to illustrate?

SUGGESTED ANSWERS

1. Answer: A.

If Z has an inverse Q , then $Z \S Q = Q \S Z =$ the identity element. So first find the identity element. By inspection it is Y , because $W \S Y = W$, $X \S Y = X$, $Y \S Y = Y$, $Z \S Y = Z$; the commutative equations also work.

(Note that values in the column under the \S are equal to the values in the column under the Y .)

Now that we know the identity element is Y , we need to find the Q such that $Z \S Q = Q \S Z = Y$. By inspection, the value that works is $Q = W$.

2. Answer: B.

$$\frac{(x/3) - (3/x)}{1 - (3/x)} = \frac{(x^2/3x) - (3 \cdot 3/3x)}{(x/x) - (3/x)} = \frac{(x^2 - 9)/3x}{(x-3)/x} = \frac{(x+3)}{3}$$

3. Answer: C.

$|f(x)| \geq$ for all x . Only Graph C is ≥ 0 for all x .

4. Answer: B.

Recall that $i^2 = -1$, $i^3 = -i$, $i^4 = +1$, $i^5 = i$, and then the cycle begins again.
 $i^5 * i^6 * i^7 * i^8 = i * i^2 * i^3 * i^4 = i * (-1) * (-i) * (+1) = + i^2 = -1$.

Equivalently: $i^5 * i^6 * i^7 * i^8 = i^{26} = i^{24} * i^2 = (i^4)^6 * i^2 = (+1)^6 * (-1) = -1$.

5. Answer: D.

If you notice that each y value is half of the prior y value, you will choose D as the only equation which does that.

Alternately: The slopes are not constant, so A. is false. All three remaining equations give $y=10$ at $x=1$, and they also give $y=5$ at $x=2$. You can continue plugging in $x=3$ and $x=4$, and it will not be until $x=4$ that only the last equation still works. Rather than plugging in $x=3$ and $x=4$, you might skip ahead and plug in $x=5$, where only the last equation still works.

6. Answer: D.

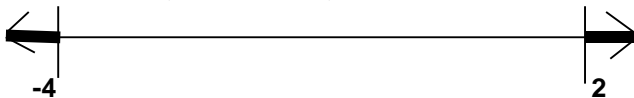
$0.5\% = .005$ A decrease of 0.5% per month requires multiplying each month by $(1-.005) = .995$. $250000 * (.995^6) = 242593$

7. Answer: B.

$|3x + 3| \geq 9$ means $+(3x + 3) \geq 9$ or $-(3x + 3) \geq 9$.

$+(3x + 3) \geq 9$, $3x \geq 6$, $x \geq 2$

$-(3x + 3) \geq 9$, $-3x \geq 12$, $x \leq -4$.



8. Answer: A.

$V = (1/3) (\text{area of base}) (\text{height})$. Area of base = $8*8=64$.

The slant height S is the hypotenuse of a right triangle with legs $h = \text{height of pyramid}$, and $r = \text{radius of a circle inscribed in the square base}$.

$S^2 = h^2 + r^2$, $5^2 = h^2 + (8/2)^2$, $h = 3$.

$V = (1/3) (64) (3) = 64$.

9. Answer: B.

$4^{|x+1|} + 3 = 19$, $4^{|x+1|} = 16 = 4^2$, $|x + 1| = 2$,

$+(x+1) = 2$, $x = +1$; and $-(x+1) = 2$, $x = -3$.

10.

A. The number of ways 5 things can be chosen from 10 things without regard to order is a combinations problem: $nCr = n! / [(n-r)! * r!]$. $10C5 = 10! / [5! * 5!] = 252$

B. You would expect the mean of the sample means to be the same as the population mean, 5.5. You would expect the standard deviation of the sampled means to equal the standard deviation of the population, divided by the square root of the sample size $n = 5$: $2.9/\sqrt{5} = 2.9/2.236 = 1.3$ (rounded). Note that it is NOT $2.9/\sqrt{100} = 2.9/10 = .29$

C.

$$(5+ 4+ 3+ 8+ 9) / 5 = 5.8$$

$$(10+ 2+ 7+ 4+ 8) / 5 = 6.2$$

$$(7+ 10+ 5+ 1+ 4) / 5 = 5.4$$

$$(9+ 2+ 3+ 7+ 5) / 5 = 5.2$$

$$(1+ 8+ 6+ 2+ 10) / 5 = 5.4$$

Mean of the sample means: $(5.8 + 6.2 + 5.4 + 5.2 + 5.4) / 5 = 5.6$

Sample SD equals: The sum of the squares of the deviations of each point from the mean; divided by one fewer than the number of points; then take the square root.

$$\sqrt{ [(5.8 - 5.6)^2 + (6.2 - 5.6)^2 + (5.4 - 5.6)^2 + (5.2 - 5.6)^2 + (5.4 - 5.6)^2] / 4 } \\ = \sqrt{ [.77 / 4] } = \sqrt{.1925} = .44 \text{ rounded.}$$

(The division by 4 is not a mistake; a sample standard deviation has an "(n-1)" term in the divisor, not an "n" term; this is called the adjustment for bias, and is discussed in any statistics text.)

D. Several possible answers: One is that the SD of the sampled means is considerably smaller than the SD of the population (.44 versus 2.9). Another is that if the various sample means were graphed, they would look like approximately a bell-shaped (normal) curve, even though the original distribution was not bell-shaped.

(Note: The grading on the essay questions is generous regarding partial credit; they are more interested in your logic and explanations, than in your arithmetic.)

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